Standing Waves in Strings

**Purpose**

The general appearance of waves can be shown by means of standing waves in a string. This type of wave is very important because most of the vibrations of extended bodies, such as the prongs of a tuning fork or the strings of a piano, are standing waves. The purpose of this experiment is to study how the speed of the wave in a vibrating string is affected by the stretching force and the frequency of the wave.

**Theory**

Standing waves (stationary waves) are produced by the interference of two traveling waves, both of which have the same wavelength, speed and amplitude, but travel in opposite directions through the same medium. The necessary conditions for the production of standing waves can be met in the case of a stretched string by having waves set up by some vibrating body, reflected at the end of the string and then interfering with the oncoming waves.

A stretched string has many natural modes of vibration (three examples are shown above). If the string is fixed at both ends then there must be a node at each end. It may vibrate as a single segment, in which case the length \( L \) of the string is equal to \( \frac{1}{2} \) the wavelength \( \lambda \) of the wave. It may also vibrate in two segments with a node at each end and one node in the middle; then the wavelength is equal to the length of the string. It may also vibrate with a larger integer number of segments. In every case, the length of the string equals some integer number of half wavelengths.

If you drive a stretched string at an arbitrary frequency, you will probably not see any particular mode; many modes will be mixed together. But, if the tension and the string's length are correctly
adjusted to the frequency of the driving vibrator, one vibrational mode will occur at a much greater amplitude than the other modes. For any wave with wavelength \( \lambda \) and frequency \( f \), the speed, \( v \), is
\[
(eq. 1) \quad v = \lambda f
\]
The speed of a wave on a string is also given by
\[
(eq. 2) \quad v = \sqrt{\frac{F}{\mu}}
\]
where \( F \) is the tension in the string and \( \mu \) is the linear density (mass/length) of the string.

In this experiment, standing waves are set up in a stretched string by the vibrations of an electrically-driven String Vibrator. The arrangement of the apparatus is shown to the right. The tension in the string equals the weight of the masses suspended over the pulley. You can alter the tension by changing the masses. \( L \) is the length of the string and \( n \) is the number of segments. (Note that \( n \) is not the number of nodes). Since a segment is 1/2 wavelength then \( 2L \)
\[
(eq. 3) \quad \lambda = \frac{2L}{n}
\]

**Setup**

1. Measure the exact length of a piece of string several meters long. Measure the mass of the string and calculate the linear density, \( \mu \) (mass/length).
   (If your balance is not precise enough to measure that length of string, use a much longer piece of string to calculate the linear density.)

2. As shown in the picture, clamp the String Vibrator and pulley about 100 cm apart. Attach the string to the vibrating blade, run it over the pulley, and hang about 100 g of mass from it. Cut off the excess string.

3. Measure from the knot where the string attaches to the String Vibrator to the top of the pulley. This is distance \( L \). (\( L \) is not the total length of the string that you measured in step 1.)

4. Connect the AC power supply to the String Vibrator.
**Procedure**

1. Adjust the tension by adding to or subtracting from the hanging mass so that the string vibrates in 2 segments. Adjust the tension further to achieve a “clean” node at the center. Also check the end of the vibrating blade; the point where the string attaches should be a node. It is more important to have a good node at the blade than it is to have the largest amplitude possible. However, it is desirable to have the largest amplitude possible while keeping a good node.

2. Record the hanging mass, \( m \) (do not forget to include the mass of the hanger). How much uncertainty is there in your value? By how much can you change the hanging mass before you see an effect? Record the uncertainty.

3. Repeat for second string.

**Analysis Method 1**

1. Calculate the tension (including the uncertainty) in the string.
   
   \[ \text{Tension} = F = mg \]

2. Calculate the speed (including uncertainty) of the wave from your observed values of tension \( F \) and linear density \( \mu \).

   \[ v_{F\mu} = \sqrt{\frac{F}{\mu}} \]

   Record your calculated value with the uncertainty and the correct number of significant figures.

3. Calculate the speed from the wavelength \( \lambda \) and frequency \( f \).

   \[ v_w = \lambda f \]

   (In the U.S. \( f = 60.0 \) Hz. In most other countries \( f = 50.0 \) Hz.)

1. Compare the two values of speed. What is the difference? How does the difference compare to the uncertainty that you determined in step 2?

2. Calculate the percentage by which \( v_{F\mu} \) deviates from \( v_w \).

   \[ \% \text{Deviation} = \left( \frac{v_{F\mu} - v_w}{v_w} \right) \times 100\% \]

6. Repeat the Procedure and this analysis for standing waves of three and four segments.

**Analysis Method 2**

1. Repeat the Procedure for standing waves of 3, 4, 5, etc. segments. Get as many as you can. Record the mass, \( m \), (including uncertainty) and the number of segments, \( n \), in a table.
For every value of mass, calculate the tension (including uncertainty) in the string:

\[ Tension = F = mg \]

Make a graph of \( F \) versus \( n \). Describe in words the shape of the graph.

For every value of \( n \), calculate \( 1/n^2 \). Make a graph of \( F \) versus \( 1/n^2 \). Does the graph look linear?

Find the slope (including uncertainty) of the best fit line through this data.

Combine equations 1, 2, and 3 (from the Theory section), and show that the tension can be written as:

\[ f = \left( 4\mu f^2 L^2 \right) \left( \frac{1}{n^2} \right) \]

Thus the slope of an \( F \) versus \( 1/n^2 \) graph is \( 4\mu f^2 L^2 \).

Use the slope from your graph to calculate the density, \( \mu \), of the string. Also calculate the uncertainty of \( \mu \).

Compare this measured value of density to the accepted value. (You calculated the accepted value of \( \mu \) from the mass and length of the string). What is the difference? How does the difference compare to the uncertainty that you calculated in step 7?

Calculate the percent difference of the two values of \( \mu \).

\[ \%difference = \left( \frac{|\text{Measured}_1 - \text{Measured}_2|}{\text{Measured}_1 - \text{Measured}_2} \right) \times 100 \]

Further Investigations

1. Hang a mass on the string with a value that is about halfway between the masses that produced standing waves of 3 and 4 segments. The string should show no particular mode.

   Place a “bridge” so that you can see the exact fundamental \( (n = 1) \) between the String Vibrator and the bride. What is the wavelength?

   Slide the bridge away from String Vibrator until the string vibrates in 2 segments. How does the wavelength of the two-segment wave compare to the wavelength of the previous one-segment wave?

   Why is a standing wave created only when the bridge is at certain locations? What are these locations called?

2. If a strobe is available, observe the standing wave on a string with the strobe light. Draw a diagram explaining the motion of the string.
String 1:
Total Length of the String, \( L = \) _______________ Mass of the String, \( m = \) _______________

Mass per unit Length, \( \mu = m/L = \) _______________

Frequency of Vibrator

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<thead>
<tr>
<th>Number of Loops, ( n )</th>
<th>Hanging mass ( (\ ) )</th>
<th>Tension, ( T ) ( (\ ) )</th>
<th>Length of ( n ) loops ( (\ ) )</th>
<th>Length of one Loop ( (\ ) )</th>
<th>Wavelength ( \lambda ) ( (\ ) )</th>
<th>( \sqrt{T} ) ( (\ ) )</th>
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String 2:

Total Length of the String, $L =$ .......................... Mass of the String, $m =$ ..........................

Mass per unit Length, $\mu = m / L =$ ..........................

Frequency of Vibrator

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