Projectile Motion & Conservation of Energy

Equipment

Qty	Item	Part Number
1	Mini Launcher	ME-6800
1	Metal Sphere Projectile	
	1 and 2 Meter Sticks	
1	Large Metal Rod	ME-8741
1	Small Metal Rod	ME-8736
1	Support Base	ME-9355
1	Plumb Bob	SE-8728
1	Double Rod Clamp	ME-9873
1	Carbon Paper	
	White Paper	

Purpose

The purpose of this activity is to examine some of the basic behaviors and properties of simple projectile motion. Among those properties, and behaviors that will be examined are how does the initial angle at launch affect the range of the projectile, and to see that the mechanical energy of a simple projectile is conserved.

Theory: Projectile Motion

Projectile motion is a form of motion in which an object (called the projectile) is launched at an initial angle ϑ, with an initial velocity v_i. Which the projectile is in flight the only force acting on it is the force of gravity (we are ignoring any air resistance). Since, near the Earth's surface, the force of gravity causes masses to be accelerated downwards at a constant rate of $g = 9.81\ {}^{m}/_{s^2}$ we can use our simple Kinematic equations to describe projectile motion. Using the standard coordinate system where the x-direction is purely vertical, and the y-direction is purely vertical we obtain the following equations of projectile motion for the y-direction;

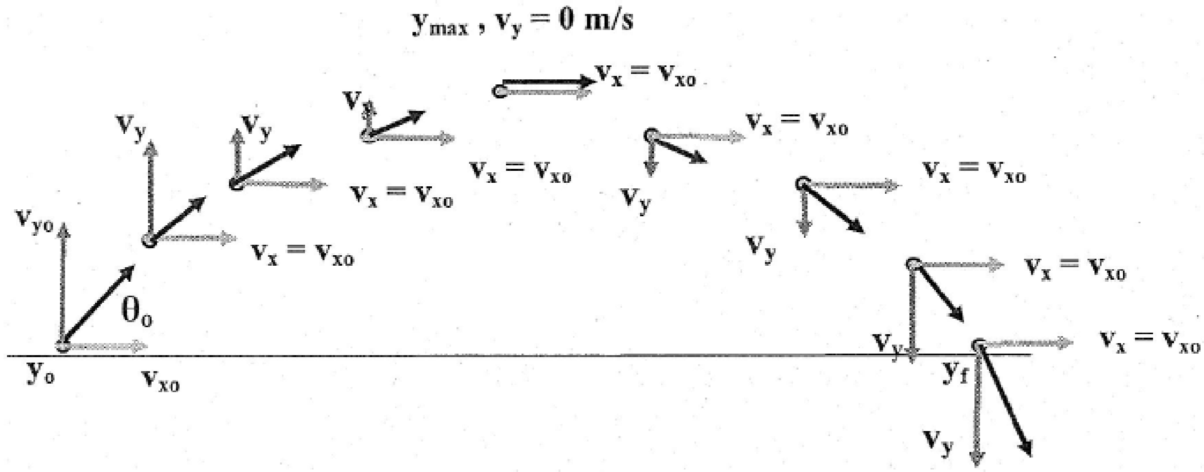
$$y = y_i + v_{iy}t - \frac{1}{2}gt^2 \qquad y = \frac{1}{2}\left(v_y - v_{iy}\right)t \qquad v_y = v_{iy} - gt \qquad v_y^2 = v_{iy}^2 - 2g\Delta y$$

Since gravity acts purely in the vertical direction, and we have no other force acting on the projectile during flight, the acceleration in the x-direction is zero $a_x = 0$. That results in our kinematic equations in the x-direction reducing to the following;

$$x = x_i + v_x t \qquad v_x = constant$$

Assuming that the initial angle ϑ is measured from the horizontal then the projectiles velocity components are given by;

$$v_x = v_i \cos(\vartheta) \quad v_y = v_i \sin(\vartheta)$$



From the above diagram we can see the behaviors of the velocity vector, and its components, of a projectile while it is in flight. The x-component is constant through the entire flight. While the y-component is constantly changing. The y-component is equal to zero when the projectile is at its maximum height, and therefore the velocity vector is at its minimum value when the projectile is at its maximum height.

The x-displacement, $\Delta x = x - x_i$, the projectile goes through during its flight is called 'the range' of the projectile. One of the things we will look at in this activity is how changing the initial launch angles affects the range of the projectile.

Conservation of Energy

The total mechanical energy of a ball during projectile motion is its potential energy (*PE*), and its kinetic energy (*KE*). (Here we are ignoring any rotation of the ball) In the absence of air resistance, or any other non-conservative force, the total energy will be conserved. When a ball is shot straight up, its initial *PE* is usually to be zero since the initial height is normally defined to be equal to zero, $PE_i = mgy_i = 0$. This makes all of the ball's energy associated with its motion, i.e. $KE_i = \frac{1}{2}mv_i^2$. As the ball flies upward the force gravity acts on the ball accelerating it downwards. Since gravity is a conservative force it will not change the amount of energy the ball possesses, but will cause the energy to transform from Kinetic to Potential. As the ball flies upwards its velocity will be constantly decreasing, again due to the force of gravity accelerating the ball downwards, till eventually the velocity will be zero, $v_f = 0$. At this point the ball will stop moving upwards, therefore the ball will have reached its maximum (final) height, y_f, and the ball's Kinetic to Potential will be given by, $PE_f = mgy_f$, and $KE_f = \frac{1}{2}mv_f^2 = 0$. Since the only force acting on the mass (the ball) during this whole process is the force of gravity, the by Conservation of Energy we can set the sum of the initial energies equal to the sum of the final energies, giving us;

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$
$$\frac{1}{2}mv_i^2 = mgy_f$$

So we see that the initial Kinetic energy equals the final potential energy for our simple projectile.

Procedure: Determining Initial Velocity

1. Attach the Large Metal Rod to the Support Base, and then put the support base on the ground.
2. Then use the Double Rod Clamp, and the Small Metal Rod to attach the Mini-Launcher to the Large Metal Rod.
3. Use a meter stick to make sure that the bottom of the barrel opening of the Mini-Launcher is 1 meter above the floor. Record this as y_i for **TABLE 1**.
4. Using the protractor on the side of the Mini-Launcher set the initial launch angle to 0^o. This will result in the initial velocity being purely in the in x-direction, and therefore the initial y-component of the velocity will be zero.

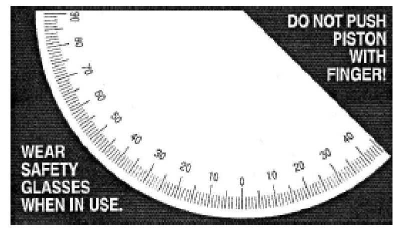


Illustration 2: The mini launcher is equipped with a built-in protractor and plumb bob so as to launch from specific angles.

5. Now dangle the Plump Bob right next to the Mini-Launcher such that its string crosses the little sign at the center of the white circle on its side, and the mass just barely touches the floor. With a pencil put a little mark on the floor where the Plumb Bob is touching it. This is the initial x-coordinate of the center of mass of the projectile at the moment it will leave the Mini-Launcher.
6. Place a large object about 2 to 3 meters in front of the Mini-Launcher. (One of your book bags, or something similar will do fine) This will serve as a barrier to stop the projectile.
7. Insert the Metal Sphere Projectile into the barrel of the Mini-Launcher, then using a pencil or pen push it back into the barrel until you hear a click. The Mini-Launcher is now at setting 1. (There are 3 settings)

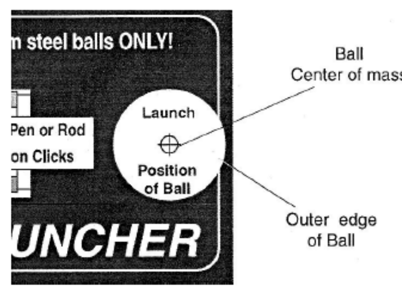


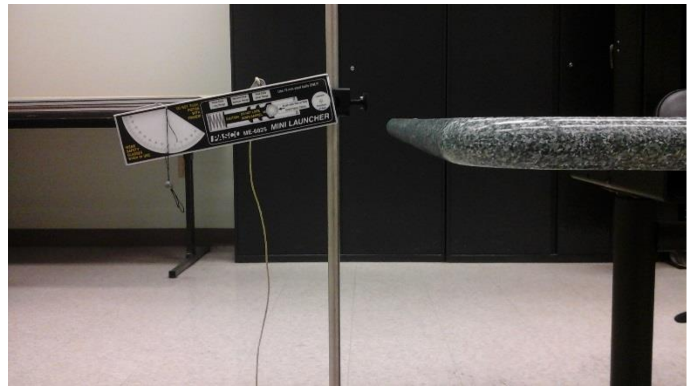
Illustration 1: When performing measurements such as range and launch height, always measure from the crosshatch, ie, the 'Ball Center of Mass' as shown in this picture.

8. Pull on the little rope attached to the Mini-Launcher's trigger to fire the projectile.
9. Note about where the projectile hit the floor, and tape a white piece of paper at that location.
10. Now on top of the white piece of paper place a piece of carbon paper with the carbon side (the dark/black side) facing downward. **DO NOT TAPE DOWN THE CARBON PAPER.**
11. Now shot the projectile 5 times onto the carbon paper, then remove the carbon paper. There should now be 5 black marks on the white piece of paper signifying the locations that the projectile hit the floor. (These 5 marks should be closely packed together. If they are not, you need to make sure everything is still aligned correctly, and then try again.)
12. Using meter stick(s) measure the displacements from the projectile's initial x-coordinate, and the 5 marks on the white paper. Record these x-displacements in **Table 1**, for setting 1.
13. Repeat steps 6 – 12 for settings 2, and then settings 3. (setting 2 = 2 clicks, setting 3 = 3 clicks)

Projectile Motions on an Uneven Surface

1. Place a large object about 2 to 3 meters in front of the Mini-Launcher. (One of your book bags, or something similar will do fine) This will serve as a barrier to stop the projectile.
2. Using the protractor on the side of the Mini-Launcher set the initial launch angle to 10^o, and make sure the bottom of the barrel opening of the Mini-Launcher is 1 meter above the floor. Record this as y_i for **TABLE 2**.
3. Insert the Metal Sphere Projectile into the barrel of the Mini-Launcher, then using a pencil or pen push it back into the barrel until you hear a click. The Mini-Launcher is now at setting 1.
4. Pull on the little rope attached to the Mini-Launcher's trigger to fire the projectile.
5. Note about where the projectile hit the floor, and tape a white piece of paper at that location.
6. Now on top of the white piece of paper place a piece of carbon paper with the carbon side (the dark/black side) facing downward. **DO NOT TAPE DOWN THE CARBON PAPER.**
7. Now shot the projectile 5 times onto the carbon paper, then remove the carbon paper. There should now be 5 black marks on the white piece of paper signifying the locations that the projectile hit the floor. (These 5 marks should be closely packed together. If they are not, you need to make sure everything is still aligned correctly, and then try again.)
8. Using meter stick(s) measure the displacements from the projectile's initial x-coordinate, and the 5 marks on the white paper. Record these x-displacements in **Table 2**.
9. Using the protractor on the side, reset the initial launch angle to 20^o, and repeat procedure, making sure that the height of the Mini-Launcher such that the bottom of the barrel opening stays at 1 meter.
10. Then repeat again for all the angles listed in **Table 2**.

Projectile Motion on an even plane



1. Reposition your Mini-Launcher so that it is now right next too, aimed down the length of your lab table. Adjust the height of the Mini-Launcher such that the bottom of the barrel opening is at the same height as the table top. This means that the initial height and final height of the projectile will be the same, and therefore the y-displacement is zero. Record $\Delta y = 0$ for **Table 3**.
2. Using the protractor on the side of the Mini-Launcher set the initial launch angle to 10^o.
3. Place a large object about 1 to 2 meters in front of the Mini-Launcher. (One of your book bags, or something similar will do fine) This will serve as a barrier to stop the projectile.
4. Insert the Metal Sphere Projectile into the barrel of the Mini-Launcher, then using a pencil or pen push it back into the barrel until you hear a click. The Mini-Launcher is now at setting 1.
5. Pull on the little rope attached to the Mini-Launcher's trigger to fire the projectile.
6. Note about where the projectile hit the table top, and tape a white piece of paper at that location.
7. Now on top of the white piece of paper place a piece of carbon paper with the carbon side (the dark/black side) facing downward. **DO NOT TAPE DOWN THE CARBON PAPER.**
8. Now shot the projectile 5 times onto the carbon paper, then remove the carbon paper. There should now be 5 black marks on the white piece of paper signifying the locations that the projectile hit the table top. (These 5 marks should be closely packed together. If they are not, you need to make sure everything is still aligned correctly, and then try again.)
9. Using a meter stick measure the displacements from the projectiles initial x-coordinate (which will be the little sign at the center of the white circle on the Mini-Launcher's side) and the 5 marks on the white paper. Record this these x-displacements in **Table 3**.
10. Using the protractor on the side, reset the initial launch angle to 20^o, and repeat procedure, making sure that the height of the Mini-Launcher such that the bottom of the barrel opening is at the same height as the table top.
11. Then repeat again for all the angles listed in **Table 3**.

Conservation of Mechanical Energy of a Projectile

1. Reposition the Mini-Launcher so that it is just about as far down the Large Metal Poll it can go without the mass hanging from the protractor touching the ground.
2. Using the protractor on its side set the launch angle to 90°. The Mini-Launcher should be aimed Straight upwards.
3. Place a meter stick, vertically, right next to the Mini-Launcher. Measure the height of the Center of Mass Mark on the side of the Mini-Launcher, and record this height as y_i for **Table 4**.
4. Insert the Metal Sphere Projectile into the barrel of the Mini-Launcher, then using a pencil or pen push it back into the barrel until you hear a click. The Mini-Launcher is now at setting 1.
5. Pull on the little rope attached to the Mini-Launcher's trigger to fire the projectile. The projectile will fly straight upwards. Repeat this a few times till you have a good idea how high the projectile is going to rise. Then with one of your group's members positioned with their eye about that level fire the projectile again. This member should now be able to read off the height the rises too. (Since we are measuring the initial height from the center of the ball, we need to measure the final height from the center of the ball too.) Record this height in **Table 4**.
6. Repeat till you have 5 measurements for setting 1.
7. Repeat procedure for settings 2, and 3. For setting 3 you will most likely need a 2 meter stick.
8. Weight the ball, and record its mass in **Table 4**.

Analysis

Table 1 $y_i =$ ______

	Setting 1	**Setting 2**	**Setting 3**
$\boldsymbol{\Delta x_1}$			
$\boldsymbol{\Delta x_2}$			
$\boldsymbol{\Delta x_3}$			
$\boldsymbol{\Delta x_4}$			
$\boldsymbol{\Delta x_5}$			
$\boldsymbol{\Delta x_{avg}}$			
$\boldsymbol{t}$			
$\boldsymbol{v_i}$			

1. Calculate the average x-displacement for each setting, and enter your answers in **Table 1**.

2. Using the equation $y = y_i + v_{iy}t - \frac{1}{2}gt^2$ calculate the time of flight for each setting, and enter your answers in **Table 1**.

3. Using the equation $v_i = \frac{\Delta x}{t}$ calculate the initial velocity for each setting, and enter your answers in **Table 1**.

Table 2: Uneven surface $\Delta y = 0$

	10°	20°	30°	40°	45°	50°	60°	70°	80°
Δx_1									
Δx_2									
Δx_3									
Δx_4									
Δx_5									
Δx_{avg}									

4. Calculate the average x-displacement for each setting, and enter your answers in **Table 2**. Which angle gives the longest range?

5. Graph Range vs. Initial Launch Angle.
6. Using the equation $y = y_i + v_{iy}t - \frac{1}{2}gt^2$ calculate the time of flight for the initial launch angle of 45°.

7. Using equation $\Delta x = v_{ix}t$ calculate the theoretical range for your projectile with the initial launch angle of 45°.

8. Calculate the % difference between your measured range, and your theoretical range for the initial launch angle of 45°.

Table 3: Even Plane $y_i =$ _______

	10°	20°	30°	40°	45°	50°	60°	70°	80°
Δx_1									
Δx_2									
Δx_3									
Δx_4									
Δx_5									
Δx_{avg}									

9. Calculate the average x-displacement for each setting, and enter your answers in **Table 2**. Which angle gives the longest range?

10. Graph Range vs. Initial Launch Angle. Do you notice a symmetry in the graph? If so, what is that symmetry? Does the same symmetry exist for the graph on the uneven surface? Write your answer on the graph the for an even plane.
11. Using the equation $y = y_i + v_{iy}t - \frac{1}{2}gt^2$ calculate the time of flight for the initial launch angle of 45°.

12. Using equation $\Delta x = v_{ix}t$ calculate the theoretical range for your projectile with the initial launch angle of 45°.

13. Calculate the % difference between your measured range, and your theoretical range for the initial launch angle of 45°.

Table 4: Conservation of Mechanical Energy $y_i =$ _______ $m =$ _________

	Setting 1	Setting 2	Setting 3
y_{f1}			
y_{f2}			
y_{f3}			
y_{f4}			
y_{f5}			
y_{f-avg}			
PE_f			
v_i			
KE_i			

14. Calculate the average final height for each setting, and the Potential Energy the projectile has at its maximum height. Then record those answers in **Table 4**.

15. Transfer the values for the velocities for each setting from **Table 1** to **Table 4**, then calculate the initial Kinetic Energies for each setting. Record those answers in **Table 4**.

16. Calculate the % difference between the initial Kinetic Energy, and the final Potential Energy for each setting. Was the mechanical energy conserved for each setting?