Rotational Inertia

Introduction

Recall that an object in rotational motion can be described by both linear and angular quantities:

Quantities	Linear	Angular	Relationship
Displacement	x (m)	θ (rad)	$x = r\,\theta$
Velocity	v (m/s)	ω (rad/s)	$v = r\,\omega$
Acceleration	a (m/s^2)	α (rad/s^2)	$a = r\,\alpha$

In the Centripetal Force lab it was shown that a centripetal force is required to keep an object in rotational motion where angular velocity is constant. To produce angular acceleration, t*orque* is required. Torque is the rotational analogue to Force and is defined by the Greek letter Tau (τ):
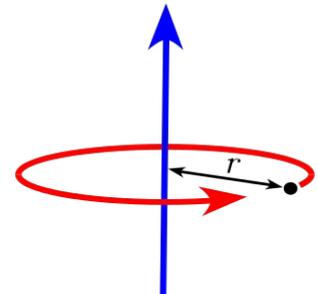
$$\tau = r \times F$$

where r is the radius of the circular path of a point mass and F is the applied force. If the Force is tangential to motion, ie, perpendicular to the radius, then

$$\tau = r\,F_T$$

Using Newton's 2nd law and the above relations, we find that:

$$\tau = (m a_T)r = m(r\alpha)r = (mr^2)\alpha = I\alpha$$

Where I is the *moment of inertia*, the rotational counterpart to mass. It is important to note that the moment of inertia is a property of the object under investigation; it does *not* depend on torque or angular acceleration. These two quantities, no matter how much they change, will not affect the moment of inertia in any way. The moment of inertia is dependent only upon mass, distribution of mass, and the shape of the object (particularly its radius). Listed below are the moments of inertia for the two objects used in this lab. They have uniform mass distribution and are rotating about the z-axis:
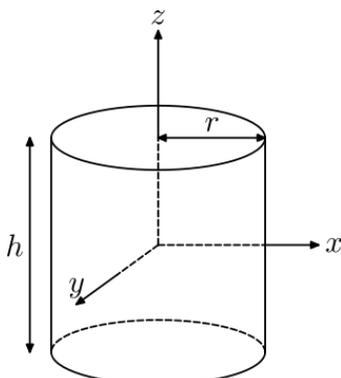
Solid Disk:

$$I = \frac{1}{2}mr^2$$

Ring:

$$I = \frac{1}{2}m(r_1^2 + r_2^2)$$
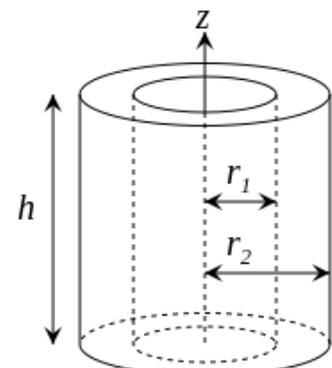
Illustration 2: Solid Disk

Illustration 1: Ring

Goal

To investigate the moment of inertia of two different objects, a solid disk and a ring.

Equipment

Rotary Motion Sensor	Mass and Hanger Set
Base and Support Rod	String
Solid Disk	Ring

Lab Notes

In this lab, you will determine the experimental moments of inertia for a solid disk and a ring by applying a torque to rotate the object under investigation. This will be accomplished by placing the object atop the rotary motion sensor, and using a hanging mass (m_H) to rotate the sensor (step) pulley.
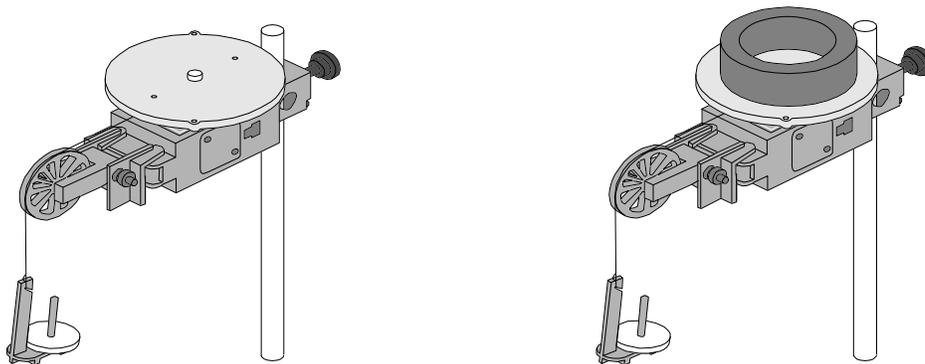

Illustration 3: Left: Setup for Solid Disk; Right: Setup for Solid Disk + Ring

As you can see, the mass hanger is attached to a string which travels over a secondary pulley and is connected to the step pulley which is not visible since is is beneath the disk. Note that the moment of inertia of the ring will not be determined independently of the disk. The disk is attached to the step pulley by a thumbscrew and the 2 tabs on the ring will sit in the holes of on the disk.
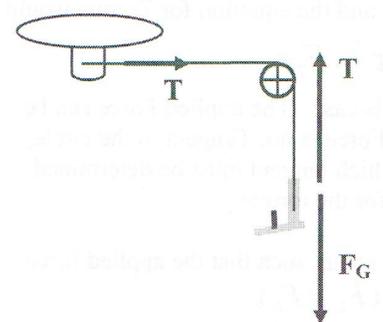
From the free body diagram, we find that:

$$T - m_H g = m_H(-a_T) \quad \text{or} \quad T = m_H(g - a_T)$$

And this tension is exactly the tangential force that turns the step pulley:

$$\tau = r_P F_T = r_P m_H(g - a_T) \quad \text{where } r_P \text{ is the radius of the step pulley.}$$

Thus, since $\tau = I\alpha$ and $\alpha = a_T/r_P$, the experimental moment of inertia is:
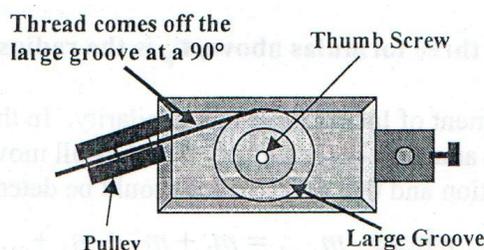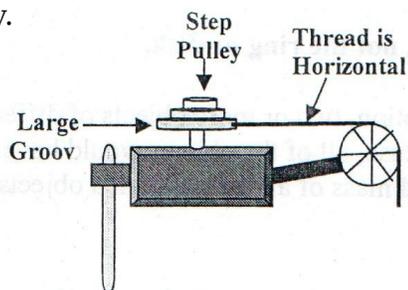
$$I_{\exp} = \frac{r_P m_H(g - a_T)}{\alpha} = r_P^2 m_H\left(\frac{g}{a_T} - 1\right)$$

Setup

Pulley System

IMPORTANT: Since the applied force is tangential, the secondary pulley must be properly aligned to the step pulley.

1. Attach a mass hanger with a 20g mass to the string and wind the string up using the large groove of the step pulley.
2. The string must come off the large groove of the step pulley at 90º and be perfectly aligned to the secondary pulley. Ensure that the string exits the step pulley and enters the secondary pulley tangentially as shown in the previous image. This will require you to set the secondary pulley at an angle. Once this is set, the direction of rotation can not be changed.
3. The string must be horizontal between the point it leaves the step pulley and the point it enters the secondary pulley. Ensure also that the string is horizontal between the step and secondary pulley as shown in the previous image.
4. Take measurements of the radius of: Step Pulley (r_P), Ring, inner radius (R_1), Ring, outer radius (R_2), and Disk (R) and record in Table 1.
5. Take measurements of the mass of the Ring (M_R) and the Disk (M_D) and record in Table 1.

Computer

1. Log in, open Capstone, then click 'Hardware Setup':

2. Click the Digital Channel 1 button on the interface picture and add 'Rotary Motion Sensor'. Set the frequency to 20 hz.

3. Click the 'Data Summary' tab:

   (a) Click the Gear icon to the right of 'Rotary Motion Sensor'.
       i. Ensure 'Resolution' is set to 'Low: 360 counts per revolution'
       ii. Ensure 'Linear Accessory' is set to 'Large Pulley (Groove)', then click 'Ok'
   (b) Click 'Velocity (m/s)' to highlight, then click the Gear icon to the right of it.
   (c) Click 'Numerical Format' and change the 'Number of Decimal Places' to 3, then click 'Ok'.

4. Click 'Graph' on the right hand pane and drag it into the center of the workspace.

5. On the graph, click 'Select Measurement' on the y-axis and choose 'Velocity'. You should now have a Force vs. Time graph.

Helpful Hints:

1. Recall that the upper jaws of the vernier caliper are designed to find inner radius.
2. Always wind the string around the step pulley in the same direction.
3. The direction of rotation determines the sign of velocity and thus a_T, so use the absolute value.
4. Ensure mass hanger does not swing on its way down.
5. The mass hanger is 5g, so don't forget to include this in m_H.
6. All theoretical calculations utilize the dimensions of the actual objects under investigation.
7. All experimental calculations utilize the dimensions of the step pulley and mass hanger.

Procedure

Objective 1: Disk

1. Wind up the mass hanger with 20g of mass on it (m_H = 25g).
2. Start recording, then release mass.
3. Stop recording just before the mass reaches the end of its travel. You should have a linear v vs t graph. If the mass reached the end of its travel and started winding back up, highlight only the linear portion of the graph applicable to the downward travel of the mass.
4. Click the fit button to obtain the slope of the line, ie, tangential acceleration (a_T).

5. Record this value in Table 2.
6. Repeat procedure 4 more times, adding 5g to the mass hanger each time.

Objective 2: Disk + Ring

7. Add the ring to the disk. Ensure the 2 tabs on the ring fit snugly into the holes on the ring so the ring does not dislodge during motion.
8. Repeat the procedure as for the disk, starting with 25g and record in Table 3.

Calculations

1. For both tables, calculate τ, α, and I_{exp}, then average I_{exp}.
2. For Table 2, calculate the theoretical moment of inertia of the disk, then calculate %Error.
3. For Table 3, subtract the average I_{exp} of the disk from the average I_{exp} of the disk + ring. This will give you the average I_{exp} of the ring.
4. Calculate the theoretical moment of inertia of the ring, then calculate %Error.

Data

Table 1:

Item	Value
Step Pulley Radius (r_P)	
Disk Radius (R_D)	
Disk Mass (M_D)	
Ring Inner Radius (R_1)	
Ring Outer Radius (R_2)	
Ring Mass (M_R)	

Table 2: Data, Disc

Mass	a_T (m/s²)	τ (Nm)	α (rad/s²)	I_{exp} (Nms²)
25g				
30g				
35g				
40g				
45g				
			Average I_{exp}	
			$I_{theoretical}$	
			%Error	

Table 3: Data, Disc + Ring

Mass	a_T (m/s²)	τ (Nm)	α (rad/s²)	I_{exp} (Nms²)
25g				
30g				
35g				
40g				
45g				
I_{exp} (Disc + Ring) − I_{exp} (Disc) = I_{exp} (Ring)			Average I_{exp}	
			Avg. I_{exp} Ring	
			$I_{theoretical}$	
			%Error	

Analysis

1. Give an example of an object or device that depends on rotational inertia.

2. Explain possible sources of error that would account for the difference between experimental and theoretical moments of inertia.

3. For Table 2, Graph τ vs α and find the slope.

   (a) What does the slope of the graph represent?

   (b) Find the %Error between the slope of the graph and the quantity it represents.

4. Use dimensional analysis to show that the units of the moment of inertia can also be kgm^2.

Sample Calculations: