# Uniformly Accelerated Motion in 1D

Introduction

An object with zero acceleration $\vec{a}=0$ is said to be in uniform motion. An object in uniformly accelerated motion has non-zero but *constant* acceleration, $\vec{a}=a_0$. Acceleration represents how velocity changes with time ($\Delta v/\Delta t$), and velocity represents how position changes with time ($\Delta x/\Delta t$). In one dimension, we set position to be in the positive x-direction. This will provide the following kinematic equations that will describe this type of motion:

Base Equation	Uniformly Accelerated Motion
Linear: y = mx + b	$\vec{a}=a_0$
Linear: y = mx + b	$\vec{v}=\vec{a}t+v_0$
Quadratic Polynomial: y = Ax² + Bx + C	$\vec{x}=\dfrac{1}{2}\vec{a}t^2+v_0t+x_0$

Note the relationship between the kinematic equations and their base equation such that their graphical representation can be easily obtained:
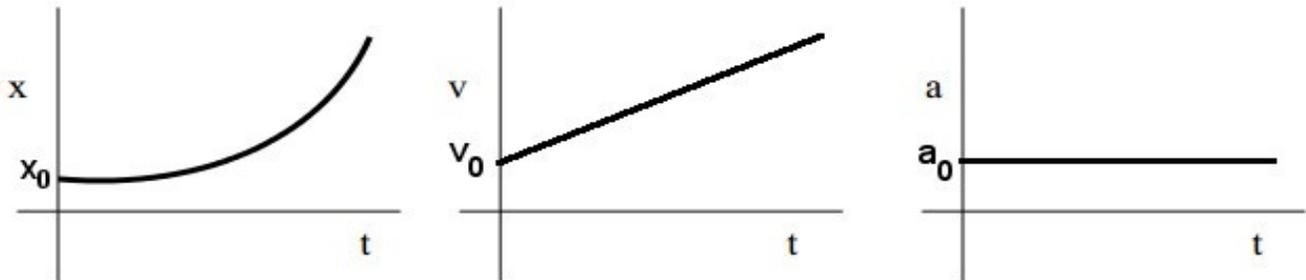
As you can see, acceleration vs time is a linear graph with no slope, ie, m (slope) = 0 and b (y-intercept) = a_0. Velocity vs time is also linear where m (slope) = *acceleration* and b (y-intercept) = v_0. For position vs time, the quadratic coefficient A = ½ a, B = v_0 and C = x_0. Thus, the coefficients of the corresponding base equation represent a physical quantity.

Furthermore, acceleration can be determined from any of the 3 graphs. For acceleration vs time it is simply the y-intercept and for velocity vs time, it is simply the slope since:

$$a_{avg}=\bar{a}=\frac{\Delta v}{\Delta t}=\frac{v_f-v_i}{t_f-t_i}$$

Average vs Instantaneous Velocity

What if we wish to find velocity from position vs time graphically? Here, we pick any 2 points on the graph, then take the slope of the line that connects them. This will provide the average velocity between those 2 points. The *instantaneous* velocity is determined by the the same procedure as for average velocity, with the exception that the 2 position points are chosen to be very very (*infinitesimally*) close to each other.

Goal

To study motion with constant acceleration and learn the relationships between position, velocity, acceleration and time for an object in uniformly acceleration motion (UAM).

Equipment

Fan Cart	Motion Sensor
Track	

Lab Notes

In this lab, the fan attached to the cart will provide the force necessary for the cart to undergo UAM, and the motion sensor will track the cart's position along the track which will be logged by the computer program Capstone. Capstone will also calculate the velocity and acceleration of the cart such that all 3 graphs will be created and viewable in real time as the motion occurs.

Propagation of Error

It is important to note the error associated with these calculations. Whatever error exists in the motion sensor will propagate to the velocity calculation and again when acceleration is calculated. This is called *propagation of error*. Thus, the acceleration vs time graph will tend to be *noisy*. That is, the graph will not be very smooth like the position and velocity graphs. You should attempt to obtain smooth position and velocity graphs, but do not attempt to obtain a smooth acceleration graph (for better visual appearance, ie more visible linear progression, increase the scale of the y-axis).

To Obtain Good Data

Things that can cause noise in the position and velocity graphs include debris in the track or on the wheels, motion sensor settings and interference, thus:

1. Always clean cart wheels and track before starting the run.
2. Ensure the cart is at least 20cm away from the motion sensor before starting the run.
3. Adjust the motion sensor as necessary. You may need to make multiple adjustments and runs before you get smooth graphs. If you have difficulty, contact your TA.

Motion Sensor Settings

The motion sensor has a toggle switch on it such that one can adjust to wide or narrow beam, and typically, wide beam works the best for this experiment. In addition, the eye is elevation adjustable with marked settings. The greatest interference occurs when the eye is at 0, or perpendicular to the motion of the cart. For best results, adjust the eye slightly off 0, either clockwise or counter-clockwise.

<u>Data Fitting</u>

One of the main objectives of this lab is *data fitting*, that is, applying an appropriate fit to your data. For example, it has been discussed that the position vs time graph should be quadratic, thus, an appropriate fit is a quadratic curve. Capstone will provide a measure of how well this curve 'fits' your data; it is called *mean squared error* and the closer this value is to zero, the better the fit. For linear graphs such as velocity and acceleration, the measure of fit is the r-value; the closer this value is to 1, the better the fit.

<u>Acceleration Error</u>

You may also notice that the acceleration vs time graph, although noisy, is not constant as it should be; it has a small slope. This quantity is called 'Jerk', defined by the change in acceleration with respect to time ($\Delta a/\Delta t$), and is a notable phenomenon to anyone who drives or rides in a vehicle. For example, if you are at a stop light that turns green and you press the accelerator too quickly you will notice that your head and parts of your body get 'jerked' backward.

In this experiment, jerk should be very small and will simply be considered systematic error in the experiment. In the analysis, you will be prompted to provide educated guesses on what causes this.

Setup

1. Log in, open Capstone, then click 'Hardware Setup':

2. Click the Digital Channel 1 button on the interface picture and add 'Motion Sensor'. Set the frequency to 20 hz.

3. Click the 'Data Summary' tab:

   1. Click 'Position' to highlight, then click the Gear icon to the right of it.
   2. Click 'Numerical Format' and change the 'Number of Decimal Places' to 3, then click 'Ok'.
   3. Repeat for 'Velocity' and 'Acceleration'.

4. Click 'Graph' on the right hand pane and drag it into the center of the workspace. You can do this again to show 2 graphs side by side, or open new pages for the other graphs (recommended).

5. For one graph, click 'Select Measurement' on the y-axis and choose 'Position'. You should now have a Position vs. Time graph.
6. For the 2nd graph, click 'Select Measurement' on the y-axis and choose 'Velocity'. You should now have a Velocity vs. Time graph.
7. For the 3rd graph, click 'Select Measurement' on the y-axis and choose 'Acceleration'. You should now have an Acceleration vs. Time graph.

Procedure

<u>Recording Data</u>

1. After cleaning the track, cart wheels, and adjusting the photo eye, place the fan cart about 20 cm from the motion sensor.
2. Hold the cart and turn the fan on.
3. Click 'Record', this will turn on the motion sensor and start the timer.
4. Release the fan cart and let it travel down the track, and stop recording just before it reaches the end.
5. Turn off the fan and check the position and velocity graphs for smoothness.
6. If necessary, adjust photo eye and repeat procedure to obtain smooth graphs.

<u>Objective 1: Instantaneous/Average Velocity</u>

1. On the position vs time graph, zoom in on 2 consecutive points. Click and drag the mouse across a small set of points to highlight them and then click 'auto-scale'. Repeat until you have just 2 consecutive points.

2. Use the 'Coordinates' tool to determine the 2 data points and record them in Table 1.
3. Highlight the points then click the drop down arrow next to the 'fit' button and choose 'Linear' to apply a linear fit to these 2 points and record the slope in Table 1. Remember that the slope here is velocity, so don't forget to include units.
4. Go to the velocity vs time graph and using the 'Coordinates' tool, find the point with the closest velocity value corresponding to this slope. Record the data point in Table 1.

<u>Objective 2: Data Fitting</u>

1. Go to position vs. time, de-select the 2 points and click 'auto-scale' to show the full graph.
2. Click the drop down arrow next to the 'fit' button and choose 'Quadratic' to apply a quadratic fit to your data.
3. Record the ABC coefficients in Table 2
4. Write the quadratic equation that corresponds to your graph in Table 2.
5. Go to velocity vs. time, de-select any points if necessary and 'auto-scale'.
6. Click the drop down arrow next to the 'fit' button and choose 'Linear' to apply a linear fit to your data.
7. Record the slope and y-intercept in Table 2 then write the linear equation that corresponds to your graph.
8. Repeat for acceleration vs time.
9. Export the coordinate data for each graph to an excel file for post-lab graphing.

Data

Table 1

Position vs Time	Result
Data Point 1 (x, t)	
Data Point 2 (x, t)	
Slope of Data Point Fit	
Velocity vs Time	
Data Point (v, t)	

Table 2

Position vs Time	Result
Mean Squared Error (MSE)	
ABC Coefficients	
Quadratic equation of fit	x =
Velocity vs Time	
r-value	
Slope & y-intercept	
Linear equation of fit	v =
Acceleration vs Time	
r-value	
Slope & y-intercept	
Linear equation of fit	a =

Analysis

1. Using Microsoft Excel or similar graphing program, re-create the 3 graphs. Use the trendline function to fit each graph and include the equation & r-value. Don't forget to include Titles and Axes Labels with units.

2. Using the data from Table 1:
   (a) At what time did the average velocity occur?

   (b) How does this time correspond to the times from the 2 points selected in the position vs time graph?

3. What are the appropriate units for the <u>slope</u> of the:

   (a) Position vs Time graph?

   (b) Velocity vs Time graph?

4. For <u>Position</u> vs Time data:

   (a) How well does your data correspond to the quadratic fit? (Hint: Use mean squared error)

   (b) Did your quadratic fit of this graph provide initial position? If yes, what is its value?

   (c) Did your quadratic fit of this graph provide initial velocity? If yes, what is its value?

   (d) Did your quadratic fit of this graph provide acceleration? If yes, what is its value?

5. For <u>Velocity</u> vs Time data:

   (a) How well does your data correspond to the linear fit? (Hint: Use the r-value)

(b) Did your linear fit of this graph provide initial position? If yes, what is its value?

(c) Did your linear fit of this graph provide initial velocity? If yes, what is its value?

(d) Did your linear fit of this graph provide acceleration? If yes, what is its value?

6. For <u>Acceleration</u> vs Time data:

   (a) How well does your data correspond to the linear fit? (Hint: Use the r-value)

   (b) Did your linear fit of this graph provide initial position? If yes, what is its value?

   (c) Did your linear fit of this graph provide initial velocity? If yes, what is its value?

   (d) Did your linear fit of this graph provide acceleration? If yes, what is its value?

   (e) Did your linear fit of this graph yield Jerk? If yes, what is its value?

   (f) If Jerk was observed, was its value small enough to be neglected, ie, close to zero?

   (g) If Jerk was observed, what experimental factors could cause this error? (Remember, human error is not an acceptable answer.)