Simple Harmonic Motion

Introduction

Simple harmonic motion is a special type of *periodic* motion, such that an object will always follow the same path and at some point return to its initial position; it will take the same amount of time to make each round trip. Each round trip is called a *cycle*, and the amount of time it takes the object to complete one cycle is called the *period*, T. The reciprocal of the period is *frequency f*, the number of cycles per unit time.

Two conditions must be met for an object to be in simple harmonic motion:

1. The *restoring force*, the force trying to return the object back to equilibrium, must be proportional to the displacement, but in the opposite direction. The farther away an object is displaced, the greater the restoring force.

2. The period must be independent of the initial displacement. No matter how far away an object is displaced, it always takes the same amount of time to make one cycle and return to that same point.

One example of this motion is the simple pendulum; a mass m, connected to a rod or string of length l. However, the displacement angle must be small, otherwise the period will become dependent on the angle. Another example is a mass m, connected to a spring with *spring constant* k. In each case, the mass is displaced from equilibrium and released. The mass then travels through the equilibrium point to an extremum, stops changes direction and travels back through equilibrium to the other extremum, stops and changes direction again to return to equilibrium and so on.

Today's lab will focus on the second example which is governed by *Hooke's Law*, F = -kx. Thus, the restoring force F, is proportional to the displacement x, and the negative sign represents the fact that the restoring force is opposite of the displacement. The conditions for simple harmonic motion are met.

From Newton's 2nd Law, we can find that the acceleration is $a_x = -(k/m)x$. Recall that the relationship between linear and rotational acceleration is $a = r\omega^2$. Since $x = r\cos\theta$, and the motion is linear, x = r, in this case. Furthermore, $\omega = 2\pi f$. Thus,

$$\omega=\sqrt{\frac{k}{m}} \quad \text{and} \quad T=2\pi\sqrt{\frac{m}{k}}$$

Food for thought: Since k is a measure of spring stiffness, which spring in the collisions lab had a high, and which had a low, k?

Goal

To study simple harmonic motion in a mass spring system.

Equipment

Force Sensor	Mass and Hanger Set
Motion Sensor	Spring
Base and Support Rod	String

Lab Notes

In this lab, you will experimentally determine the spring constant of a spring, and use this to find the period of a mass spring system. Thus, this lab has two parts.

Part 1: Here, you will hang a mass hanger from a spring attached to the force sensor. By slowly adding mass to the hanger, and measuring the displacement associated with a specific force, the spring can be determined.

Part 2: Here, the mass will be constant, but you will use both the force sensor and the motion sensor to track the movement of an oscillating mass on a spring to experimentally determine its period in two ways. The period can be determined utilizing k and the equation from the introduction, and, from the data obtained by the motion sensor.

Setup

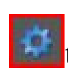
Mass-Spring System

1. Hang a spring and mass hanger from the force sensor
2. Tare the force sensor.

Computer

1. Log in, open Capstone, then click 'Hardware Setup': 

2. Click the Analog Channel A button on the interface picture and add 'Force Sensor'. Set the frequency to 20 hz.
3. Click the Digital Channel 1 button on the interface picture and add 'Motion Sensor'. Set the frequency to 50hz.
4. Click the 'Data Summary' tab: 

   (a) Click 'Force (N)' to highlight, then click the Gear icon to  the right of it.
   (b) Click 'Numerical Format' and change the 'Number of Decimal Places' to 3, then click 'Ok'.
   (c) Click 'Time (s)' to highlight, then click the Gear icon to the right of it.
   (d) Click 'Numerical Format' and change the 'Number of Decimal Places' to 3, then click 'Ok'.

5. Click the 'Table & Graph" icon.

   (a) Table

      i. In the x-coordinate column, click 'Select Measurement'
      ii. Click 'Create New' then 'User-Entered Data'
      iii. Label the highlighted term 'User Data' as 'Stretch'
      iv. Hit Enter, then label the highlighted term 'Units' as 'm' for meters.
      v. In the y-coordinate column, click 'Select Measurement'
      vi. Click 'Force'

   (b) Graph

      i. For the y-axis, click 'Select Measurement'
      ii. Click 'Force', this will provide a Force vs Time graph

6. At the bottom of the screen, change 

 to 

Helpful Hints:

1. Do NOT overload the spring. This will change its spring constant.

Procedure

<u>Objective 1: Determine Spring Constant</u>

1. Measure the distance from the tabletop to the bottom of the hanger. Record this value in Table 1, and also as the 1st x-coordinate data point in the table in Capstone.
2. Click 'Preview', the timer will start. 

3. Observe the F vs t graph and ensure the Force is zero. If not, tare the force sensor again. If the force sensor will not tare, contact your TA.
4. Press 'Keep Sample'. This will provide your first data point in the table. 

5. On the graph's x-axis, click the label 'Time' and change to 'Stretch'. You will see this first data point.
6. Add 20g to the mass hanger.
7. Wait for the mass hanger to stabilize, then click 'Keep Sample'.
8. Measure the distance from the tabletop to the bottom of the mass hanger. Record this value in Table 1, and also as the 2nd x-coordinate data point in the table in Capstone.
9. Repeat # 6-8 five more times, but using 10g increments.
10. Click 'Stop'.
11. Click 'Fit'  and select 'Linear'. Record the slope in Table 1.
12. From this slope, calculate the spring constant.
13. Export data to excel to re-graph for your lab report.

Objective 2: Determine Period

14. Remove mass from previous objective and add enough mass to the hanger so that the spring's stretched length is 6-7 times its unloaded length. Record value in Table 2. No more than 70g. Do NOT overload the spring, this will change its spring constant.
15. Place the motion sensor directly beneath the mass hanger.
16. Ensure that the mass hanger will not come closer than 20cm to the motion sensor when oscillating.
17. Open a position vs time graph.
18. Practice oscillating the spring with the mass. Movement should stay vertical.
19. Start the timer and track the oscillations on the graph. It should be a smooth damped sinusoidal curve. If the curve is not smooth, check the distance between the motion sensor and the mass hanger. Or, check the motion sensor eye alignment or beam setting.
20. When you have about 10 smooth oscillations, stop the timer. Or, restart timer and oscillations if necessary.
21. Highlight these oscillations and autoscale the graph. 
22. Using the  coordinates tool, find the time at the 1st peak and record in Table 2.
23. Move the coordinates tool to the next 6 peaks and record their times.
24. Find the Period of oscillation for each of the 6 cycles.
25. Average the Periods.

Data

<u>Table 1</u>

Mass (g)	Distance	Stretch/Displacement (m)
0		
20		
30		
40		
50		
60		
70		

Spring Constant:______________________

<u>Table 2</u>

<table>
<tr><td>Mass (kg)</td><td></td><td colspan="6"></td></tr>
<tr><td>Peak</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>Time (s)</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Period (s)</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Average Period: _________________________

Sample Calculations:

Analysis

1. Using the spring constant from Objective 1, and the mass from Objective 2, calculate the expected period of oscillation.

2. Compare the expected period of oscillation from question 1 with the average period of oscillation from Objective 2 using %Difference.

3. What reasons would account for the %Difference? Speculate on sources of error.

4. If the position of the mass is farthest from the equilibrium position, what is the magnitude of the mass' velocity?

5. If the position of the mass is at the equilibrium position, what is the magnitude of the mass' velocity?