**Projectile Motion**

**Introduction**

Projectile motion is a form of motion in which an object (called a projectile) is launched at an angle $\theta$, with initial velocity $v_0$, that moves along a parabolic path. Thus, the motion is in two dimensions and is described by the kinematic equations for uniformly accelerated motion:

$$
\begin{align*}
    x &= \frac{1}{2} a_x t^2 + v_{0x} t + x_0 \\
    v_x &= a_x t + v_{0x} \\
    a_x &= a_{0x} \\
    y &= \frac{1}{2} a_y t^2 + v_{0y} t + y_0 \\
    v_y &= a_y t + v_{0y} \\
    a_y &= a_{0y}
\end{align*}
$$

However, since gravity acts only in the vertical direction and we assume the projectile has no mechanism to accelerate, the acceleration in the x-direction equals zero, and in the y-direction equals gravity:

$$
\begin{align*}
    x &= v_{0x} t + x_0 \\
    v_x &= v_{0x} = \text{Constant}! \\
    a_x &= 0 \\
    y &= \frac{1}{2} g t^2 + v_{0y} t + y_0 \\
    v_y &= -g t + v_{0y} \\
    a_y &= -9.80 \text{m/s}^2
\end{align*}
$$

Note that velocity in the x-direction is constant, but not zero. Also note that the initial velocity has been split into its x and y components, $v_{0x}$ and $v_{0y}$, and remember that the components of the velocity vector are:

$$
v_x = v \cos \theta \quad \text{and} \quad v_y = v \sin \theta
$$

If we move initial position from both equations to the left side, we can define Range (how far the projectile traveled or will travel), and launch height (if launch and landing positions are different):

$$
\begin{align*}
    \text{Range} &= \Delta x = v_0 \cos \theta \ t \\
    v_x &= v_0 \cos \theta = \text{Constant}! \\
    a_x &= 0 \\
    \text{Launch Height} &= \Delta y = \frac{-1}{2} g t^2 + v_0 \sin \theta \ t \\
    v_y &= -g t + v_0 \sin \theta \\
    a_y &= -9.80 \text{m/s}^2
\end{align*}
$$

The time of flight of a projectile can be determined using the quadratic formula which will be derived in the post lab analysis.
Goal

Given this theory, the goal of today's lab is two-fold:

- The first task is to determine the initial velocity of a projectile shot from a mini-launcher.
- The second task is determine how the range depends upon the launch angle, and at which angle maximum range is achieved for two cases. In case 1 the initial and final heights are the same (shooting on the table), and in case 2, the heights are not the same (shooting off the table).

Equipment

- Plumb bob
- Meter stick
- Carbon paper
- White paper
- C-Clamp and bracket
- Mini Launcher and Steel ball

Safety!

Projectile shooting can be somewhat difficult and potentially dangerous in a confined environment. Please follow these safety rules to not only protect yourself, your lab mates, but also the equipment.

- Always be aware of your surroundings and where you are shooting.
- Do NOT shoot at anyone, especially your TA. Malicious shooting is grounds for dismissal.
- Do NOT shoot towards computer equipment, especially monitors.
- As with any firing weapon, do NOT dry fire, that is do NOT pull release without projectile.
- As with any firing weapon, do NOT look down the barrel.
- Only depress piston to the 1st click.
- Do NOT use your finger to depress the piston.
- Always have someone watching where the ball lands to stop it from rolling about the room.
Helpful Hints

- Do a 'test' fire to approximate where the projectile will land.
- BEWARE: If launcher is not properly secure, the action of 'pulling the trigger' can move the launcher from its set angle. This will negatively impact your data and you will have to redo.
- An open binder standing on edge works great to stop the ball.
- Dividing, and then alternating roles, is recommended, ie, 1 person 'fires', 1 person keeps an 'eye on the ball', 1 person measures, and 1 person records.

Procedure

**Objective I** – Determine the Initial Velocity of the Projectile. *Shooting off the Table*

Initial velocity can be determined by launching the ball horizontally (0 degrees) off of the table and measuring the vertical and horizontal distance through which the ball travels. Then, the initial velocity can be used to calculate where the ball will land when the ball is shot an angle.

1. Use the c-clamp and bracket to clamp the mini-launcher to the edge of the lab table.
2. Set the mini-launcher to 0°, to shoot horizontally.
3. Use the plumb bob to determine the spot on the floor (x₀) directly below the launcher crosshatch. See Illustration 5.
4. Measure from the crosshatch on the launcher to x₀ on the floor to determine Δy (FYI: it is negative, since y_f is lower than y_0) and record in Table 1.
5. Insert the ball into the launcher and use a pencil to depress piston to 1-click.
6. Test fire the ball to approximate its landing location.
7. Secure a piece of white paper to the landing zone and place a piece of carbon paper over it.
8. Measure the distance from x₀ to the leading edge of the secured paper. (having this value will save time measuring range) Let's call this value the 'paper distance', or x_p. Record this value in Table 1.
9. Fire the ball; when it lands on the carbon paper, it will make a mark on the white paper.
10. Measure the distance from the leading edge of the paper to the center of the mark. Let's call this the 'Trial Distance', or x_T. This is the value you will record in Table 1 for Trial 1.
11. Repeat # 9 and 10, four more times for a total of 5 trials.
12. Average the 5 trial distances, and record as 'Average Distance'
13. Add this value to x_p (the paper distance), to find Δx, the 'Total Average Range'.
14. Using the kinematic equations: use Δy to find t, the time of flight (Remember sin(0°) = 0 for ). Then use t to find v_0 (Remember cos(0°) = 1).
Objective II – Determine how the range depends upon the launch angle for 2 cases.

Not only is range affected by the launch angle, it is also affected by whether the ball’s final position is at the same elevation as its initial position. If it is the same elevation, \( \Delta y = 0 \). If it is not, \( \Delta y \) will affect the angle that produces the maximum range of the ball. The angle that produces the maximum range for level surface shooting is 45°, whereas the angle for non-level shooting is 30°.

Note: For both cases, range should follow a parabolic trend, that is, range will increase as angle increases until the angle that produces the maximum range. Then, range will decrease and angle increases. If this does not occur, you must redo one or more of the angles.

Case 1 – Range vs. Angle, Shooting off the table

1. Adjust the angle of the launcher to 20°, measure \( \Delta y \) and record in Table 2.
2. Test fire, secure white paper, measure \( x_P \) and record in Table 2.
3. Fire again and measure \( x_T \) in Table 2 for Trial 1.
4. Repeat # 3 for 4 more trials.
5. Average the trial distances, add to \( x_P \) to find Total Average Range.
6. Repeat # 1-5 for each additional angle in Table 2. Note: Changing angle can change \( \Delta y \) & \( x_P \).

Case 2 – Range vs. Angle, Shooting on the table

1. Reset the launcher so that the crosshatch is at the same level as the table to fire on the table at 20° (Use illustration 4 as an example). It is imperative that the level is the same, \( \Delta y \) must equal zero or you will have error in your data.
2. Test fire, secure white paper, measure \( x_P \) and record in Table 3.
3. Fire again and measure \( x_T \) in Table 3 for Trial 1.
4. Repeat # 3 for 4 more trials.
5. Average the trial distances, add to \( x_P \) to find Total Average Range.
6. Repeat # 1-5 for each additional angle in Table 3.
Table 1 – Determine the Initial Velocity of the Projectile.

<table>
<thead>
<tr>
<th>Vertical Height (Δy)</th>
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<td>Trial Number</td>
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<td>Average Distance</td>
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<td>Paper Distance (xₚ)</td>
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<td>Total Average Range</td>
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<tr>
<td>Time of Flight (t)</td>
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<td>Initial Velocity (v₀)</td>
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</table>

Show Calculations:
Table 2 – Range vs. Angle, *Shooting off the table*

<table>
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<tr>
<th>Angle</th>
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Table 2 – Range vs. Angle, *Shooting on the table*

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Analysis

1. For data tables 2 and 3, plot (by hand) Range vs. Angle and draw a parabolic fit. Do not connect the dots.

2. From the graphs, what angle gives the maximum range for each case: on and off the table?

3. Which case provides the greater angle for maximum range?

4. Which case provides the larger maximum range, and why?

5. Using the initial velocity determined in Table 1, and the angle that provided the maximum range from Table 2, calculate the theoretical range from the kinematic equations. **Hint:** You must use the quadratic formula to solve for t since \( \sin \theta \neq 0 \). Solve analytically first so you can use the same equation in question 7.

6. Using percent difference, compare the value from question 5 to the Total Average Range for that angle from Table 2.

7. Using the initial velocity determined in Table 1, and the angle that provided the maximum range from Table 3, calculate the theoretical range from the kinematic equations. **Hint:** Use the equation you found for t from question 5, where \( \Delta y = 0 \).

8. Using percent difference, compare the value from question 7 to the Total Average Range for that angle from Table 3.