

EXAMPLES OF ERROR PROPAGATION FOR SPECIFIC EXPERIMENTS

Ohm's Law & Resistors

Problem: The parallel combination of three resistors R_1 , R_2 , and R_3 is written as:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Where $R_1 = \bar{R}_1 \pm \Delta R_1$, $R_2 = \bar{R}_2 \pm \Delta R_2$ and $R_3 = \bar{R}_3 \pm \Delta R_3$ are the absolute uncertainties for those resistors. Determine the uncertainty in R .

Answer:

The equation for absolute uncertainties for R is:

$$\Delta\left(\frac{1}{R}\right) = \left[\left(\frac{\partial\left(\frac{1}{R}\right)}{\partial R_1} \Delta R_1 \right)^2 + \left(\frac{\partial\left(\frac{1}{R}\right)}{\partial R_2} \Delta R_2 \right)^2 + \left(\frac{\partial\left(\frac{1}{R}\right)}{\partial R_3} \Delta R_3 \right)^2 \right]^{\frac{1}{2}} \quad \text{Equation AE1}$$

$$\frac{\partial\left(\frac{1}{R}\right)}{\partial R_1} = \frac{\partial}{\partial R_1} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = -\frac{1}{R_1^2} = -\frac{1}{\bar{R}_1^2}$$

Similarly, $\frac{\partial\left(\frac{1}{R}\right)}{\partial R_2} = -\frac{1}{\bar{R}_2^2}$ and $\frac{\partial\left(\frac{1}{R}\right)}{\partial R_3} = -\frac{1}{\bar{R}_3^2}$

Subbing these values into equation AE1, we have:

$$\Delta\left(\frac{1}{R}\right) = \left[\left(\frac{1}{\bar{R}_1^2} \Delta R_1 \right)^2 + \left(\frac{1}{\bar{R}_2^2} \Delta R_2 \right)^2 + \left(\frac{1}{\bar{R}_3^2} \Delta R_3 \right)^2 \right]^{\frac{1}{2}} = \Delta K \quad \text{Equation AE2}$$

Let $\left(\frac{1}{R}\right) = k \rightarrow R = \left(\frac{1}{k}\right)$

$$\Delta R = \sqrt{\left(\frac{\partial R}{\partial k} \Delta k \right)^2}$$

$$\frac{\partial R}{\partial k} = \frac{\partial}{\partial k} \left(\frac{1}{k} \right) = -\frac{1}{k^2} \rightarrow -\frac{1}{\bar{k}^2}$$

$$\Delta R = \sqrt{\left(-\frac{1}{\bar{k}^2} \Delta k \right)^2} = \Delta k \cdot \bar{R}^2 \quad \text{Equation AE3}$$

From Equation AE2 and AE3:

$$\Delta R = \left[\left(\frac{1}{\bar{R}_1^2} \Delta R_1 \right)^2 + \left(\frac{1}{\bar{R}_2^2} \Delta R_2 \right)^2 + \left(\frac{1}{\bar{R}_3^2} \Delta R_3 \right)^2 \right]^{\frac{1}{2}} \cdot \bar{R}^2$$

$$\frac{\Delta R}{\bar{R}} = \left[\left(\frac{\bar{R}}{\bar{R}_1^2} \Delta R_1 \right)^2 + \left(\frac{\bar{R}}{\bar{R}_2^2} \Delta R_2 \right)^2 + \left(\frac{\bar{R}}{\bar{R}_3^2} \Delta R_3 \right)^2 \right]^{\frac{1}{2}}$$